# COLLISIONLESS RECONNECTION IN MAGNETOHYDRODYNAMIC AND KINETIC TURBULENCE

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## ABSTRACT

It has recently been proposed (Loureiro & Boldyrev 2017; Mallet et al. 2017) that the inertial interval in magnetohydrodynamic (MHD) turbulence is terminated at small scales not by a Kolmogorov-like dissipation region, but rather by a new sub-inertial interval mediated by tearing instability. However, many astrophysical plasmas are nearly collisionless so that the MHD approximation is not applicable to turbulence at small scales. In this paper, we propose the extension of the theory of reconnection-mediated turbulence to plasmas which are so weakly collisional that the reconnection occurring in the turbulent eddies is caused by electron inertia rather than by resistivity. We find that the transition scale to reconnection-mediated turbulence depends on the plasma beta and on the assumptions of the plasma turbulence model. However, in all cases analyzed, the energy spectra in the reconnection-mediated interval range from $E(k_\perp)dk_\perp \propto k_\perp^{-8/3}dk_\perp$ to $E(k_\perp)dk_\perp \propto k_\perp^{-3}dk_\perp$.

*Keywords:* magnetic fields — magnetohydrodynamics — turbulence — kinetic magnetic reconnection

## 1. INTRODUCTION

In many astrophysical flows, such as those governing stellar coronae and winds, the dynamics of planetary magnetospheres, the structures in the warm interstellar medium, and many others, the dissipation scales are so small that the flows become turbulent over a broad range of scales. At scales much larger than the plasma microscales (the particles' gyroradii and skin depths), the dynamics can be reasonably well approximated by single-fluid magnetohydrodynamics (MHD) (e.g., Biskamp 2003; Davidson 2017). Some fundamental processes of plasma energetics, such as plasma heating, particle acceleration, magnetic field reconnection, however, depend on plasma turbulence at the kinetic scales (e.g., Kulsrud 2005; Chen 2016). At such scales, the MHD description is not adequate, and the need arises of extending the relatively well-developed theory of MHD turbulence to the small, kinetic scales.

Recently, it has been suggested that magnetic reconnection may be a critical element of MHD turbulence (Loureiro & Boldyrev 2017; Mallet et al. 2017; Boldyrev & Loureiro 2017). This realization arises from the observation, motivated by the dynamic alignment picture (Boldyrev 2006), that turbulent eddies become progressively more susceptible to the tearing mode instability (which drives reconnection) as their characteristic scale, $\lambda$, gets smaller. Those authors have thus proposed that the role of reconnection in MHD turbulence can be quantified by comparing the characteristic timescales of the two processes. A typical turbulent eddy at scale $\lambda$ lasts an amount of time (the eddy turnover time) that is itself a function of $\lambda$. Similarly, a reconnection event at scale $\lambda$ occurs on a timescale which depends in some nontrivial way on $\lambda$. If the time associated with reconnection decreases with the scale of the eddy faster than the eddy turnover time does, then, given a large enough inertial interval, reconnection inevitably becomes important below a certain critical scale $\lambda_{cr}$. This scale is thus defined as the scale below which reconnection becomes faster than the turbulence — for this is the only way to ensure that reconnection has time to occur before the eddies are destroyed by the turbulence.

In (Loureiro & Boldyrev 2017) it was proposed that the scale-invariant energy cascade should persist in the reconnection-mediated interval, and that the energy should have a power-law Fourier spectrum. This concept was further developed in (Boldyrev & Loureiro 2017) and along somewhat different lines of reasoning in (Mallet et al. 2017).[1] However, these interesting new developments cannot be directly applied to very weakly-collisional turbulent astrophysical environments such as those mentioned earlier. There are two reasons for that. First, even if the eddy scale itself is larger than the kinetic scales, the tearing instability in such an eddy leads to the formation of finer scales (a boundary layer) which are, almost inevitably, in the kinetic regime. Indeed, for realistic values of the resistivity, one finds that the mechanism breaking the frozen flux condition, and thus enabling reconnection, is likely to be the electron inertia, rather than the resistivity. Second, turbulence in very weakly-collisionless plasmas extends to sub-ion scales; it is possible, as we will show, that reconnection may only become important at those scales (Cerri & Califano 2017; Franci et al. 2017).

This paper presents the first attempt at extending these recent ideas on the role of reconnection in turbulence to accommodate kinetic physics.

## 2. THE TEARING MODE IN A STRONGLY MAGNETIZED, COLLISIONLESS PLASMA

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[1] Interestingly, Huang & Bhattacharjee (2016) observed a similar power-law spectrum in turbulence driven by plasmoid instability. Although in their setting the reconnecting magnetic profiles were not generated by turbulence, their numerical observations may have presented the first glimpse at the phenomenon studied in these works.

Let us begin by analyzing the case of a low-beta plasma, $m_e/m_i \ll \beta \ll 1$, where $\beta = 8\pi nT/B_0^2$ (here we implicitly assume the ion and electron temperatures to be comparable, though not necessarily equal) and $B_0$ is a large-scale uniform magnetic field whose presence we assume. Small magnetic fluctuations in the direction normal to $B_0$ will be denoted as $b$. We will first consider plasma fluctuations with typical scales $a$ such that

$$a \gg \rho_i \sim \rho_s, \qquad (1)$$

where $\rho_i$ is the ion gyroscale and $\rho_s$ is the ion acoustic scale.[2] For simplicity, and without loss of generality, we assume an hydrogen plasma, which is one of the most relevant cases in astrophysical applications. As we discuss in more detail in the following section, it is reasonable to assume that the magnetic profile of a turbulent eddy at MHD scales resembles a current sheet with thickness $a$ (e.g., Boldyrev 2006; Mallet & Schekochihin 2016).

A turbulent eddy with such a magnetic profile will reconnect if it is unstable to the tearing mode (Furth et al. 1963). In this case, the magnetic profile tends to develop a singularity characterized by an inverse scale $\Delta'$, which is a fundamental parameter of the tearing instability. The other crucial parameter, the size of inner boundary layer of the tearing mode, $\delta_{in}$, is much smaller than the scale of the reconnecting magnetic profile. Due to Eq. (1), the eddy scale $a$ belongs to the MHD regime of plasma turbulence. We, however, assume that the inner scale $\delta_{in}$ belongs to the kinetic range,

$$\delta_{in} \ll \rho_i \sim \rho_s. \qquad (2)$$

It is inequality (2) that ensures that kinetic effects (electron inertia in this case) become important at the inner scale of the tearing mode. This inequality distinguishes our kinetic theory from the MHD theory of reconnection-mediated turbulence developed in (Loureiro & Boldyrev 2017; Boldyrev & Loureiro 2017).

Under these conditions, the linear growth rate of the tearing instability of such a magnetic field configuration scales as follows (see (Zocco & Schekochihin 2011) and references therein). For low $\Delta'$ (i.e., for $\Delta'\delta_{in} \ll 1$) we have:

$$\gamma \sim k v_A d_e \rho_s \Delta'/a, \qquad (3)$$

whereas for large $\Delta'$ (i.e., $\Delta'\delta_{in} \gtrsim 1$)

$$\gamma \sim k v_A d_e^{1/3} \rho_s^{2/3}/a. \qquad (4)$$

In these expressions, $v_A$ denotes the Alfvén speed based on the reconnecting magnetic field $b_a$, $k$ is the wavenumber of the perturbation parallel to the reconnecting field, $d_e \equiv c/\omega_{pe}$ is the electron skin depth (with $\omega_{pe}$ the electron plasma frequency).[3]

The instability parameter $\Delta'$ is obtained by solving the outer region (MHD) equation. It is a function of the wavenumber $k$, but its specific scaling with $k$ (for $ka \ll 1$) depends on the functional form of the reconnecting magnetic field. For the usual Harris magnetic field profile (Harris 1962) it is $\Delta'a \sim 1/(ka)$. In the Harris profile, the field reverses direction on the scale $a$, but its plateau region is much longer than $a$ (strictly speaking, it is infinite). In the other limiting case, where the scales of the field reversal and the field plateau are comparable (say a sinusoidal profile), the scaling is $\Delta'a \sim 1/(ka)^2$. Because both scalings are, a priori, possible in a turbulent eddy, we will keep this dependence generic by assuming simply that $\Delta'a \sim 1/(ka)^n$, where $1 < n \leq 2$ is a parameter (not necessarily integer).

The usual procedure to find the wavenumber of the fastest growing tearing mode is to equate the expressions for the growth rate and solve for $k_{max}$. This gives

$$k_{max}^{(n)} \sim d_e^{2/(3n)} \rho_s^{1/(3n)} a^{-2-1/n}, \qquad (5)$$

which corresponds to the maximum growth rate

$$\gamma_{max}^{(n)} \sim v_A d_e^{(1+2/n)/3} \rho_s^{(2+1/n)/3} a^{-2-1/n}. \qquad (6)$$

For the Harris sheet-like configuration, the $n \to 1$ case, the fastest growing mode and the corresponding maximal growth rate are:

$$k_{max}^{(1)} \sim d_e^{2/3} \rho_s^{1/3}/a^2, \qquad (7)$$

$$\gamma_{max}^{(1)} \sim v_A d_e \rho_s/a^3. \qquad (8)$$

In the limiting case $n = 2$, the fastest growing mode and the associated growth rate are:

$$k_{max}^{(2)} \sim d_e^{1/3} \rho_s^{1/6} a^{-3/2}, \qquad (9)$$

$$\gamma_{max}^{(2)} \sim v_A d_e^{2/3} \rho_s^{5/6} a^{-5/2}. \qquad (10)$$

### 3. KINETIC RECONNECTION IN MHD TURBULENCE

Let us now apply these scalings in the context of MHD turbulence as described by Boldyrev (2006)[4]. Therefore, we envision a tearing-unstable magnetic profile whose characteristic scale, identified as $a$ above, is the width of the turbulent eddy, $\lambda$. Correspondingly, the length of the current sheet, $L_{cs}$, is the other field-perpendicular eddy dimension,

$$\xi \sim L(\lambda/L)^{3/4}, \qquad (11)$$

where $L$ is the outer scale of the turbulence. Likewise, the Alfvén velocity that appears in the scalings above is identified with the Alfvén velocity at scale $\lambda$, i.e.,

$$v_A \to v_{A,\lambda} \sim v_{A,0}(\lambda/L)^{1/4}, \qquad (12)$$

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[2] In studies of turbulence the eddy scale $a$ is typically denoted by $\lambda$. We however keep in this work the notation traditional for the reconnection literature wherever the results of the reconnection theory are discussed.

[3] These expressions are valid for finite ion temperature; this, however, only affects the numerical prefactors that multiply these expressions (Zocco & Schekochihin 2011), and so does not affect the order of magnitude derivations that follow. Another point worth mentioning is that throughout this paper we will only focus on $\beta \lesssim 1$ plasmas; and, in addition, the reconnecting component of the magnetic field is weaker than the guide field, as pertains to the turbulence model that we envisage. As such, pressure anisotropy effects are not expected to play an important role here.

[4] In our work we use the dynamic alignment model (Boldyrev 2006, Mallet et al. 2015, Mallet & Schekochihin 2016), which self-consistently predicts the formation of current sheets in turbulence. Current sheets are observed in direct numerical simulations, which we think lends credence to the dynamic alignment hypothesis. If future theoretical studies are to replace the dynamic alignment as it is currently understood with a different model, then consistency with numerical simulations will nonetheless require such theory to describe current sheets in turbulence. Given specific predictions for the turbulent eddy structure and turn-over time in such a theory (analogous to our Eqs. 11, 12), the arguments in this paper can be adjusted accordingly. The specific scaling predictions might then change, but not the fundamental underlying idea of our argument, which is that reconnection eventually becomes competitive with the turbulence nonlinear time.

with $v_{A,0}$ is the Alfvén velocity at the outer scale $L$. The eddy turnover time at scale $\lambda$ is

$$\tau \sim \lambda^{1/2} L^{1/2} / v_{A,0}. \tag{13}$$

The following calculations hold provided that $\lambda$ remains larger than any kinetic scales.

For the general magnetic profile, the transition scale between the inertial and reconnection-dominated intervals is given by the criterion[5]

$$\gamma_{max}^{(n)} \tau \sim 1. \tag{14}$$

This yields

$$\lambda_{cr}^{(n)}/L \sim (d_e/L)^{\frac{1}{3}\left(1+\frac{2}{n}\right)/\left(\frac{5}{4}+\frac{1}{n}\right)} (\rho_s/L)^{\frac{1}{3}\left(2+\frac{1}{n}\right)/\left(\frac{5}{4}+\frac{1}{n}\right)} \tag{15}$$

In the limiting case $n = 1$, this relation becomes[6]

$$\lambda_{cr}^{(1)}/L \sim (d_e/L)^{4/9} (\rho_s/L)^{4/9}. \tag{16}$$

This expression is valid provided that $\lambda_{cr} \gg \rho_s$, i.e.,

$$\rho_s/L \ll (m_e/m_i)^2 \beta_e^{-2}. \tag{17}$$

where we have used the definition $d_e \sim \rho_s (m_e/m_i)^{1/2} \beta_e^{-1/2}$.

For an $n = 2$ type magnetic profile, we instead obtain

$$\lambda_{cr}^{(2)}/L \sim (d_e/L)^{8/21} (\rho_s/L)^{10/21}. \tag{18}$$

The validity condition $\lambda_{cr} \gg \rho_s$ now implies

$$\rho_s/L \ll (m_e/m_i)^{4/3} \beta_e^{-4/3}. \tag{19}$$

Both limits of our model provide similar transition scales and rather weak parameter restrictions for the reconnection-mediated turbulence in a low-beta plasma. For instance, for the plasma in the solar corona, where one expects $\beta_e \sim 0.01$ at the distance of $\sim 10$ solar radii from the sun (e.g., Chandran et al. 2011), we obtain from Eq. (17):

$$\rho_s/L \ll 3 \times 10^{-3}, \tag{20}$$

while the restriction corresponding to the other limiting case (19) yields

$$\rho_s/L \ll 10^{-1}. \tag{21}$$

In both cases a Hydrogen plasma is assumed. We may therefore expect that the inertial interval of the coronal Alfvénic turbulence should transform into the reconnection-mediated interval at small scales.

#### 4. TURBULENCE SPECTRUM

To obtain the energy spectrum in the reconnection-mediated range, we proceed as in Boldyrev & Loureiro (2017). Specifically, we assume that a consequence of the tearing mode becoming nonlinear is that the eddy evolution rate ($\gamma_{nl}$) then becomes enslaved to that of the mode, i.e.,

$$\gamma_{nl} \sim \gamma_t. \tag{22}$$

[5] It is implicit in what follows, and throughout the rest of the paper, that the tearing mode that first satisfies condition (14) is the intersectional mode (5) yielded by the matching of the low and large $\Delta'$ scalings. This is, in fact, not an assumption: it can be easily checked that this is indeed true for all cases that we consider.

[6] A. Mallet, A. Schekochihin, and B. Chandran have informed us in a private communication that they have independently arrived at the same expression for the transition scale.

As in the MHD case, it is known from theoretical and numerical studies that the growth rate of the kinetic tearing mode discussed in Section 2 remains unchanged from its linear value as the mode enters the nonlinear regime (e.g., Wang & Bhattacharjee 1993; Porcelli et al. 2002) .

Thus, let us define the energy cascade rate $\epsilon = V_{A0}^3/L_0$ and assume it to be independent of scale ($\lambda$) both in the inertial and reconnection ranges. Dimensionally, we have

$$\gamma_{nl} \sim \epsilon/v_{A\lambda}^2. \tag{23}$$

Then, imposing $\gamma_{nl} \sim \gamma_t$, we obtain, for $n = 1$ type magnetic profiles,

$$v_{A\lambda} \sim \epsilon^{1/3} \lambda d_e^{-1/3} \rho_s^{-1/3}, \tag{24}$$

from which one easily finds

$$E(k_\perp) dk_\perp \sim \epsilon^{2/3} d_e^{-2/3} \rho_s^{-2/3} k_\perp^{-3} dk_\perp, \tag{25}$$

where $k_\perp \sim 1/\lambda$.

Similarly, it is easy to see that for a type $n = 2$ magnetic profile one has

$$v_{A\lambda} \sim \epsilon^{1/3} \lambda^{5/6} d_e^{-2/9} \rho_s^{-5/18}, \tag{26}$$

corresponding to the energy spectrum

$$E(k_\perp) dk_\perp \sim \epsilon^{2/3} d_e^{-4/9} \rho_s^{-5/9} k_\perp^{-8/3} dk_\perp. \tag{27}$$

According to our results, the energy spectrum of Alfvénic turbulence mediated by kinetic reconnection should therefore range from $k_\perp^{-8/3}$ to $k_\perp^{-3}$, in the scale range $\lambda_{cr} \ll k_\perp \ll \rho_s^{-1}$, with $\lambda_{cr}$ yielded by Eq. (16) or Eq. (18), as appropriate.

#### 5. ULTRALOW BETA LIMIT

The limit when plasma beta is so low that $\beta_e \ll m_e/m_i$ can similarly be considered; this is relevant for, e.g., the Earth's magnetosphere (e.g., Chaston et al. 2008).

The calculation proceeds as in the previous sections. From Zocco & Schekochihin (2011), we find that in this limit the fastest growing tearing mode wavenumber and corresponding growth rate are

$$k_{max}^{(n)} \sim d_e^{1/n} a^{-1-1/n}, \tag{28}$$

$$\gamma_{max}^{(n)} \sim d_e^{1+1/n} v_A a^{-2-1/n}. \tag{29}$$

Application of the criterion stated by Eq. (14) yields the following critical scale for reconnection onset:

$$\lambda_{cr}^{(n)}/L \sim (d_e/L)^{(1+\frac{1}{n})/(\frac{5}{4}+\frac{1}{n})}. \tag{30}$$

The two limiting cases of interest of this expression are $n = 1$, for which we find

$$\lambda_{cr}^{(1)}/L \sim (d_e/L)^{8/9}; \tag{31}$$

and $n = 2$, which yields

$$\lambda_{cr}^{(2)}/L \sim (d_e/L)^{6/7}. \tag{32}$$

The validity of this analysis requires $\lambda_{cr}^{(n)} > d_e$, which in both cases reduces to $d_e < L$, a condition that is trivially satisfied.

As in the previous section, we can compute the energy spectra in this regime assuming that the growth rate of

the tearing mode in the early nonlinear stage remains unchanged from its linear value. We obtain:

$$E^{(1)}(k_\perp)dk_\perp \sim \epsilon^{2/3}d_e^{4/3}k_\perp^{-3}dk_\perp, \tag{33}$$

$$E^{(2)}(k_\perp)dk_\perp \sim \epsilon^{2/3}d_e^{-1}k_\perp^{-8/3}dk_\perp, \tag{34}$$

valid in the scale range $\lambda_{cr} \ll k_\perp \ll d_e^{-1}$, with $\lambda_{cr}$ given by Eq. (31) or Eq. (32), respectively. We see that the spectral slopes are unaltered from the previous expressions. We also note that the transition scales (31) and (32) are always larger than the electron inertial scale $d_e$. This may be consistent with the slightly larger than $d_e$ scale of the Alfvénic spectral break observed in the Earth's magnetosphere turbulence (Chaston et al. 2008, Fig. 2).

## 6. LARGE $\beta$

Another case of interest are plasmas where $\beta_e \sim 1$. This can be considered using the approximate two-fluid tearing mode scalings derived in (Fitzpatrick & Porcelli 2004, 2007).[7] Again, the procedure is entirely similar, so we simply state the key results.

The tearing mode dispersion relations in this regime are:

$$\gamma \sim kv_A(d_i/a)\Delta' d_e, \tag{35}$$

for low $\Delta'$; and

$$\gamma \sim kv_A(d_i/a)^{3/5}(d_e/a)^{2/5}, \tag{36}$$

at large $\Delta'$. The most unstable mode and corresponding growth rate is:

$$k_{max}^{(n)} \sim d_e^{3/(5n)}d_i^{2/(5n)}a^{-1-1/n}, \tag{37}$$

$$\gamma_{max}^{(n)} \sim v_A d_e^{(3+2n)/(5n)}d_i^{(2+3n)/(5n)}a^{-2-1/n}. \tag{38}$$

Note that Eq. (38) exhibits the same dependence on $a$ as Eq. (6).

Using these relationships, we find that the critical eddy size for transition to the reconnection-mediated turbulence range is

$$\lambda_{cr}^{(n)}/L \sim (d_e/L)^{4/5(3+2n)/(4+5n)}(d_i/L)^{4/5(2+3n)/(4+5n)}, \tag{39}$$

which has the following two limiting cases:

$$\lambda_{cr}^{(1)}/L \sim (d_e/L)^{4/9}(d_i/L)^{4/9}, \tag{40}$$

valid if

$$d_i/L \ll (m_e/m_i)^2; \tag{41}$$

and

$$\lambda_{cr}^{(2)}/L \sim (d_e/L)^{2/5}(d_i/L)^{16/35}, \tag{42}$$

valid if

$$d_i/L \ll (m_e/m_i)^{7/3}. \tag{43}$$

The corresponding spectra can be easily obtained:

$$E^{(1)}(k_\perp)dk_\perp \sim \epsilon^{2/3}d_e^{-2/3}d_i^{-2/3}k_\perp^{-3}dk_\perp, \tag{44}$$

$$E^{(2)}(k_\perp)dk_\perp \sim \epsilon^{2/3}d_e^{-7/15}d_i^{-8/15}k_\perp^{-8/3}dk_\perp, \tag{45}$$

[7] The equations in (Fitzpatrick & Porcelli 2004, 2007) are formally derived in the cold-ion limit. However, usually the inclusion of finite ion temperature does not modify the scalings, only numerical prefactors; so, it is possible that the results in this section apply equally to cases where $\tau \sim 1$.

whose validity range is $\lambda_{cr}^{-1} \ll k_\perp \ll d_i^{-1}$, with $\lambda_{cr}$ given by Eq. (40) or Eq. (42). These predictions for the energy spectra exhibit the same $k_\perp$ power law indices as those obtained earlier at low $\beta$, Eqs. (25, 27) — the reason being that the growth rate of the tearing mode in this regime. Eq. (38), has the same dependence on the magnetic shear length $a$ (equivalently, $\lambda$) as Eq. (6). We however note that conditions (41) and (43) are very stringent and may, in fact, imply that in plasmas where $\beta_e \sim 1$, the transition to the reconnection range cannot happen in the MHD-scale interval.

## 7. RECONNECTION IN THE KINETIC TURBULENCE RANGE

One would now like to extend these ideas to the kinetic turbulence range, when the eddies are on sub-ion scales, i.e., $\lambda < \max(\rho_i, \rho_s)$. This is, however, nontrivial, because our understanding of kinetic-scale turbulence is much less developed than that of MHD turbulence. In particular, we are not familiar with an analytical theory that offers the kinetic equivalent of Eqs. (11–13), implying, therefore, that we cannot know whether the tendency to develop current sheets that is present in the MHD range remains true in the kinetic range. However, numerical simulations and observations (e.g., Boldyrev & Perez 2012; Wan et al. 2012; TenBarge & Howes 2013; Chen et al. 2015; Wan et al. 2016; Cerri & Califano 2017; Franci et al. 2017) do show evidence for current sheet formation at such scales, and so perhaps it is legitimate to assume that current sheets remain the fundamental units of sub-ion scales turbulence. Note that the argument that follows is completely independent of the nature of the waves that are found in the kinetic regime; indeed, no specific property of the turbulence in the kinetic range is invoked, other than its tendency to form current sheets.

Let us then assume that this is indeed so. There are two options: either the critical scale for onset of the reconnection range has been met at the MHD scales, i.e., $\lambda_{cr} > \max(\rho_i, \rho_s)$, or it has not. In the latter case, we are not able to estimate it, because there is currently no theory to describe the eddy structure at those scales. Therefore, it remains to be seen whether reconnection may become important at the sub-proton scales. If, however, this is the case, we may compute the energy spectrum, provided that Eq. (22) holds and that the tearing mode growth rate in the early nonlinear regime remains unchanged from its linear value.

We will address this question in the framework of the equations derived in (Chen & Boldyrev 2017), valid at scales below the ion Larmor radius and assuming $\beta_i \gg \beta_e$ and $\beta_e \ll 1$ (Eqs. (19-20) of that reference). Such a regime may be relevant for the solar corona (e.g., Chandran et al. 2011), hot accretion flows (e.g., Quataert 1998), collisionless shocks (e.g., Treumann 2009; Ghavamian et al. 2013; Chen & Boldyrev 2017), etc. The tearing mode calculation proceeds in the usual way; it is not hard to see that the most unstable tearing mode is such that $\Delta'\delta \sim 1$ and $\delta \sim d_e$. This allows us to find immediately that:

$$k_{max}^{(n)} \sim d_e^{1/n}a^{-1-1/n}, \tag{46}$$

$$\gamma_{max}^{(n)} \sim d_i d_e^{1/n}a^{-2-1/n}, \text{ if } \beta_i \gtrsim 1. \tag{47}$$

$$\gamma_{max}^{(n)} \sim \rho_i d_e^{1/n}a^{-2-1/n}, \text{ if } \beta_i \ll 1. \tag{48}$$

The expected energy spectra follow straightforwardly:

$$E^{(1)}(k_\perp)dk_\perp \sim \epsilon^{2/3}(m_i/m_e)^{-1/3}d_e^{-4/3}k_\perp^{-3}, \qquad (49)$$

$$E^{(2)}(k_\perp)dk_\perp \sim \epsilon^{2/3}(m_i/m_e)^{-1/3}d_e^{-1}k_\perp^{-8/3}, \qquad (50)$$

if $\beta_i \sim 1$. In the opposite case of $\beta_i \ll 1$ the above expressions for the spectrum appear multiplied by the prefactor $\beta_i^{-1/3}$. These expressions apply at at scales $\lambda_{cr}^{-1} \gg k_\perp \gg d_e^{-1}$ — but, as we mention above, this range can only be precisely quantified once a kinetic theory of turbulence is available that allows us to compute $\lambda_{cr}$.

Again, interestingly, we observe that the $k_\perp$ dependence of the spectra are the same as in all cases considered above.

## 8. DISCUSSION AND CONCLUSION

We have proposed that, in collisionless plasmas, the inertial interval of Alfvénic turbulence can cross over to a reconnection-mediated interval at scales ($\lambda_{cr}$) larger than the relevant plasma micro-scales, such as the ion-acoustic scale or the electron skin depth. We predict that depending on the parameters of the model, the magnetic energy spectrum in the reconnection-mediated interval can vary from $E(k) \propto k^{-8/3}$ to $E(k) \propto k^{-3}$, both in low beta plasmas (e.g., as the solar corona, interplanetary coronal mass ejections, planetary magnetospheres, etc. (e.g., Chen et al. 2014; Chaston et al. 2008; Bale et al. 2016)) and plasmas with $\beta \sim 1$. These spectral predictions are valid for wavenumbers such that $\lambda_{cr}^{-1} \gg k_\perp \gg \lambda_{kin}^{-1}$, where $\lambda_{kin}$ is the kinetic scale relevant for each of the different cases analyzed. Given the specific scalings for $\lambda_{cr}$ that we have derived, we observe that this scale range may be narrow in several applications of interest. However, the importance of the existence of a reconnection range goes well beyond its spectral extent, for it implies that the turbulence that arrives at the kinetic scales is qualitatively different (different alignment angle, different turn-over time) than it would be were reconnection not to occur. In other words, if $\lambda_{cr}$ satisfies the validity conditions derived in each regime, then our paper predicts that the eddies at the transition from the MHD to the kinetic range can no longer be described by the model of Boldyrev (2006) (or any Kolmogorov-like model for that matter). In the typical energy cascade picture of turbulence, this result therefore implies the presence of a *new* relevant scale parameter, in addition to the Kolmogorov scale. This new scale parameter reflects a new, reconnection-mediated regime of turbulence, which may qualitatively impact not only the physics of the transition between the Alfvénic and the kinetic regimes, but also the physics of the kinetic range itself.

We have suggested that our theory may be extended to the subproton-scale turbulence $\lambda < \rho_i$. Indeed, the turbulent fluctuations at such scales resemble current sheets (e.g., Boldyrev & Perez 2012; Wan et al. 2012; TenBarge & Howes 2013; Chen et al. 2015; Wan et al. 2016; Cerri & Califano 2017; Franci et al. 2017), although those turbulent structures (and generally turbulence at kinetic scales) are relatively less well understood than their Alfvénic counterpart. If we may assume that, similarly to the Alfvénic case, the dynamics at subproton scales are governed by the fastest growing tearing modes, the reconnection-dominated turbulence should have the same scaling $E(k) \propto k^{-8/3}$ to $E(k) \propto k^{-3}$ that we have derived for the Alfvénic case.

Interestingly, the predicted spectral scaling is very close to the spectrum of turbulence $\approx -2.8$ measured in the $\beta \sim 1$ solar wind plasma below the ion-cyclotron scale (e.g., Alexandrova et al. 2009; Kiyani et al. 2009; Chen et al. 2010a, 2012; Sahraoui et al. 2013). We should caution that the close proximity of the reconnection-mediated spectra of turbulence to the spectra derived from qualitatively different turbulence models (related to cascades of the kinetic-Alfvén or, possibly, whistler waves (Howes et al. 2008; Schekochihin et al. 2009; Chen et al. 2010b; Boldyrev & Perez 2012)), may not allow one to discern what physical mechanism is dominant based solely on measurements of the spectral exponents. Numerical simulations and observations, however, offer increasing evidence that current sheets are important dynamic players in turbulence at both MHD and kinetic scales. It thus seems conceivable to suppose that their presence affects the spectral properties of turbulence; this paper presents the first theoretical analysis of how they may do so.

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